

The standard model and the constituents of leptons and quarks

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Abstract

A complete set of postulates of the standard model of the electroweak interaction and mass generation is formulated and confirmed deriving the Lagrangian for the standard model. A massive fermion is formed by a right-handed and a left-handed elementary massless fermion, exchanging a scalar doublet. The elementary massless fermions forming leptons belong to an approximate $SU(3)$ octet. The charges are quantised due to this symmetry.

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The standard model of the electroweak interaction and mass generation, the model of Glashow, Weinberg and Salam [1], very successfully describes all experimental observations in particle physics made so far. The recent discovery of the Higgs particle at CERN by ATLAS and CMS is the final proof of the validity of the standard model. After the discoveries of neutrino oscillations and of the Higgs particle, the standard model deserves a detailed and consistent formulation.

Describing the minimal standard model in an unconventional language, in this note it is assumed that leptons and quarks are formed by particles, which are more elementary. From their properties a set of postulates of the minimal standard model is worked out. As a proof of the postulates, the well established Lagrangian of the standard model is derived.

For a definition of the notation, some essentials of Yang-Mills theory [2], on which the electroweak interaction is based, are provided in the Appendix.

In order to derive the Lagrangian for the electroweak interaction [3], it is assumed that a massive fermion doublet state ψ_i , like a neutrino for $i = 1$ or an electron for $i = 2$, is composed of constituents: A massless left-handed fermion doublet state χ_{iL} and a massless right-handed fermion singlet state ψ_{iR} exchanging a scalar doublet ϕ . These three primary elementary fields have to be chosen with the following properties:

1. The left-handed massless fermion doublet χ_L has $SU(2) \times U(1)$ symmetry and is represented by

$$\chi_L = \begin{pmatrix} a \\ b \end{pmatrix} \frac{1 - \gamma^5}{2} \psi_i, \quad (1)$$

where $\psi_i = \psi_i(x^\mu)$ is a 4-spinor and the parameters a and b are complex numbers with

$$a^* a + b^* b = 1. \quad (2)$$

According to (70), the $SU(2) \times U(1)$ interaction transformation U_χ for the interaction of χ_L with massless gauge potentials is given by

$$U_\chi = \exp \left(-i \int \left(\frac{g}{\sqrt{2}} (T_+ W_\mu^+ + T_- W_\mu^-) + g T_3 W_\mu^3 + g' Y B_\mu \right) dx^\mu \right). \quad (3)$$

A constituent χ_{iL} of a massive fermion is an eigenstate of T_3 , represented by

$$\chi_{iL} = \begin{pmatrix} a \\ b \end{pmatrix}_i \frac{1 - \gamma^5}{2} \psi_i \quad \text{with} \quad \begin{pmatrix} a \\ b \end{pmatrix}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{or} \quad \begin{pmatrix} a \\ b \end{pmatrix}_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \quad (4)$$

The interaction transformation for a state χ_{iL} is

$$U_\chi = \exp \left(-i \int \left(\frac{g}{\sqrt{2}} (T_+ W_\mu^+ + T_- W_\mu^-) + g t_3 W_\mu^3 + g' y B_\mu \right) dx^\mu \right), \quad (5)$$

where t_3 is the third component of the weak isospin and y is the weak hypercharge of χ_{iL} . The two states χ_{iL} carry the same weak hypercharge y but different weak isospin $t_3 = \pm \frac{1}{2}$.

2. The right-handed massless fermion singlet ψ_{iR} has $U(1)$ symmetry and is represented by

$$\psi_{iR} = \frac{1 + \gamma^5}{2} \psi_i. \quad (6)$$

A state ψ_{iR} carries zero weak isospin and the $U(1)$ charge $g' y_i$. The $U(1)$ interaction transformation for a state ψ_{iR} is according to (56)

$$U_\psi = \exp \left(-i \int g' y_i B_\mu dx^\mu \right). \quad (7)$$

The two states ψ_{iR} carry the same weak isospin $t_3 = 0$ but different weak hypercharges y_i .

The states χ_{iL} and ψ_{iR} with equal index i together form a massive fermion ψ_i .

The massless fermions χ_{iL} and ψ_{iR} carry electric charge. The electric charge is a linear combination of the weak charge $g t_3$ and the $U(1)$ charge, defined as the linear combination for which both states χ_{iL} and ψ_{iR} , together forming a massive fermion ψ_i , are carrying the same electric charge q_i .

Expressing the interaction transformations U_χ and U_ψ in terms of the electromagnetic potential A_μ , belonging to the electric charge q_i , and the orthogonal potential Z_μ , belonging to the generator f , results in

$$g t_3 W_\mu^3 + g' y B_\mu = f_\chi Z_\mu + q_i A_\mu, \quad g' y_i B_\mu = f_\psi Z_\mu + q_i A_\mu. \quad (8)$$

These equations are valid if the potentials and the generators are rotated by the same angle θ_W

$$\begin{aligned} Z_\mu &= \cos \theta_W W_\mu^3 - \sin \theta_W B_\mu \\ A_\mu &= \sin \theta_W W_\mu^3 + \cos \theta_W B_\mu \end{aligned} \quad (9)$$

$$\begin{aligned} f_\chi &= \cos \theta_W g t_3 - \sin \theta_W g' y, & f_\psi &= -\sin \theta_W g' y_i \\ q_i &= \sin \theta_W g t_3 + \cos \theta_W g' y = \cos \theta_W g' y_i. \end{aligned} \quad (10)$$

The electric charge is thus defined. The electroweak angle θ_W , the Weinberg angle, is given by

$$\tan \theta_W = \frac{g' y_i - g' y}{g t_3}. \quad (11)$$

Expressing the generators f in terms of q_i results in

$$f_\chi = \frac{g t_3}{\cos \theta_W} - \tan \theta_W q_i, \quad f_\psi = -\tan \theta_W q_i. \quad (12)$$

After replacing the generators f in (8) by this result, the interaction transformations U_χ and U_ψ are representing a unified description of the weak and electromagnetic interactions

$$\begin{aligned} U_\chi &= \exp \left(-i \int \left(\frac{g}{\sqrt{2}} (T_+ W_\mu^+ + T_- W_\mu^-) + \frac{g t_3}{\cos \theta_W} Z_\mu + q_i (A_\mu - \tan \theta_W Z_\mu) \right) dx^\mu \right) \\ U_\psi &= \exp \left(-i \int q_i (A_\mu - \tan \theta_W Z_\mu) dx^\mu \right). \end{aligned} \quad (13)$$

The charged gauge potentials W_μ^\pm carry the elementary electric charge $\pm e$. Due to conservation of electric charge the elementary electric charge e is therefore

$$e = q_1 - q_2 = g \sin \theta_W = g' \cos \theta_W \quad (14)$$

where in addition the $U(1)$ unit charge g' has been defined for the convenience to write the electric charge q_i given by (10) as a simple relation

$$q_i = e (t_3 + y) = e y_i. \quad (15)$$

From the observed electric charge q_i the weak hypercharge of the constituents is

$$y = -\frac{1}{2}, y_1 = 0, y_2 = -1 \text{ for leptons, } y = \frac{1}{6}, y_1 = \frac{2}{3}, y_2 = -\frac{1}{3} \text{ for quarks.} \quad (16)$$

The experimental value of the electroweak angle is [3] $\theta_W \approx 28.6^\circ$. In order to see if there is a symmetry responsible for this value of θ_W , it is instructive to plot the charges of the constituents of leptons, including the corresponding anti-particle states, and to add the electric charge as an index. For example the first generation of massive leptons is composed of the constituents

$$\begin{aligned} \nu_e : & \chi_{1L}^0, \psi_{1R}^0 & e^- : & \chi_{2L}^-, \psi_{2R}^- \\ \bar{\nu}_e : & \chi_{2R}^0, \psi_{2L}^0 & e^+ : & \chi_{1R}^+, \psi_{1L}^+ . \end{aligned} \quad (17)$$

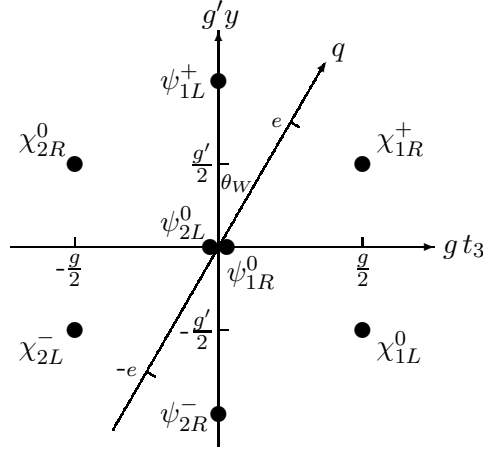


Fig.1: The charges of the massless fermions forming leptons closely approximate $SU(3)$ symmetry. A massive fermion consists of a singlet state ψ and a doublet state χ with opposite chirality, equal index $i = 1, 2$ and equal electric charge.

As Fig.1 shows, both the left-handed and the right-handed massless fermions forming leptons closely approximate a $SU(3)$ triplet and an uncharged singlet. An exact $SU(3)$ symmetry would require $\theta_W = 30^\circ$. Since θ_W slowly increases with increasing momentum transfer, there exists a momentum transfer for which the $SU(3)$ symmetry is exact.

For quarks, the $SU(3)$ symmetry is distorted due to their hypercharge $y = \frac{1}{6}$, $SU(3)$, however, requires $y = \pm \frac{1}{2}$.

Obviously, the elementary massless fermions form octets. The charges are quantised due to this symmetry.

3. In addition, a scalar field ϕ exists. The two massless fermions χ_{iL} and ψ_{iR} interact via a Yukawa interaction, mediated by the scalar potential ϕ . The Yukawa interaction is defined by the Lagrangian

$$L_Y = - \sum_i C_i \left(\bar{\psi}_{iR} \phi^\dagger \chi_{iL} + \bar{\chi}_{iL} \phi \psi_{iR} \right), \quad (18)$$

where C_i is a charge. The Lagrangian L_Y describes the reactions

$$\begin{aligned} \psi_{iR} + \phi &\rightarrow \chi_{iL} \\ \chi_{iL} &\rightarrow \psi_{iR} + \phi. \end{aligned} \quad (19)$$

These reactions conserve $U(1)$ charge and t_3 .

The Lagrangian L_Y implies that the scalar potential ϕ is a doublet. It is represented by

$$\phi = \exp(-i\Theta) \begin{pmatrix} a \\ b \end{pmatrix} H, \quad (20)$$

where $H = H(x^\mu)$ is a scalar field with no internal degrees of freedom, representing the Higgs particle [4]. Due to the conservation of $U(1)$ charge by the reactions (19) and with the help of (11) the $U(1)$ charge of ϕ is

$$g' y_\phi = g' y - g' y_i = -g t_3 \tan \theta_W, \quad (21)$$

where the same t_3 is carried by χ_{iL} and ϕ , since t_3 is conserved. Hence, the interaction transformation U_ϕ for an eigenstate of T_3 results in

$$\begin{aligned} U_\phi &= \exp \left(-i \int \left(\frac{g}{\sqrt{2}} (T_+ W_\mu^+ + T_- W_\mu^-) + g t_3 W_\mu^3 + g' y_\phi B_\mu \right) dx^\mu \right) \\ &= \exp \left(-i \int \left(\frac{g}{\sqrt{2}} (T_+ W_\mu^+ + T_- W_\mu^-) + \frac{g t_3}{\cos \theta_W} Z_\mu \right) dx^\mu \right). \end{aligned} \quad (22)$$

The doublet ϕ does not carry electric charge and does not interact with the electromagnetic potential A_μ , for the reason that χ_{iL} and ψ_{iR} carry the same electric charge.

The parameters a and b of ϕ are identical with the parameters a and b of χ_{iL} . The field $\exp(-i\Theta)$ implies a massless particle, the Goldstone boson, and represents the three remaining degrees of freedom of ϕ , where

$$\Theta = \Theta(x^\mu) = \frac{1}{\sqrt{2}} (T_+ \theta^+ + T_- \theta^-) + \frac{T_3}{\cos \theta_W} \theta^0. \quad (23)$$

The internal rotation angles θ^+, θ^- and θ^0 correspond to the three internal degrees of freedom of ϕ represented by the gauge potentials of U_ϕ .

The elementary fields and their interactions are thus defined. They imply the Lagrangian \mathcal{L} for the electroweak interaction, consisting of the following terms

$$\mathcal{L} = L_\psi + L_\chi + L_\phi + L_Y + L_W + L_B. \quad (24)$$

The terms of \mathcal{L} are

$$L_\psi = \sum_i \bar{\psi}_{iR} i \gamma^\mu (\partial_\mu - M_\psi) \psi_{iR} \quad (25)$$

$$L_\chi = \sum_i \bar{\chi}_{iL} i \gamma^\mu (\partial_\mu - M_\chi) \chi_{iL} \quad (26)$$

$$L_\phi = \frac{1}{2} \left(((\partial_\mu - M_\phi) \phi)^\dagger ((\partial^\mu - M_\phi) \phi) + \frac{m_H^2}{2} \phi^\dagger \phi - \frac{m_H^2}{4v^2} (\phi^\dagger \phi)^2 \right) \quad (27)$$

$$L_Y = - \sum_i C_i \left(\bar{\psi}_{iR} \phi^\dagger \chi_{iL} + \bar{\chi}_{iL} \phi \psi_{iR} \right) \quad (28)$$

$$L_W = -\frac{1}{4} \mathbf{W}_{\mu\nu} \mathbf{W}^{\mu\nu} \quad (29)$$

$$L_B = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu}, \quad (30)$$

where m_H is the mass of the Higgs particle, v is a constant and the interaction matrices M are due to (13) and (22)

$$\begin{aligned} M_\psi &= U_\psi^{-1} \partial_\mu U_\psi = -i q_i (A_\mu - \tan \theta_W Z_\mu) \\ M_\chi &= U_\chi^{-1} \partial_\mu U_\chi = -i \left(\frac{g}{\sqrt{2}} (T_+ W_\mu^+ + T_- W_\mu^-) + \frac{g t_3}{\cos \theta_W} Z_\mu + q_i (A_\mu - \tan \theta_W Z_\mu) \right) \\ M_\phi &= U_\phi^{-1} \partial_\mu U_\phi = -i \left(\frac{g}{\sqrt{2}} (T_+ W_\mu^+ + T_- W_\mu^-) + \frac{g t_3}{\cos \theta_W} Z_\mu \right). \end{aligned} \quad (31)$$

Hence, the complete Lagrangian of the standard model is defined. The elementary fields possess a well-defined internal symmetry and interact with massless gauge potentials, which is essential for a Yang-Mills interaction.

The Lagrangian \mathcal{L} corresponds to the diagram displayed by Fig.2 showing the constituents of a massive fermion.

A massive fermion ψ is formed by a left-handed massless fermion doublet state χ_L and a right-handed massless fermion singlet state ψ_R exchanging a scalar doublet ϕ . Thus the difference of the charges of χ_L and ψ_R is transferred. The gauge potentials emitted by χ_L and ψ_R denoted M_χ and M_ψ interact with the scalar ϕ and finally obtain a longitudinal component and mass. By the ϕ -exchange the constituents χ_L and ψ_R are interchanged but the massive fermion ψ remains unchanged.

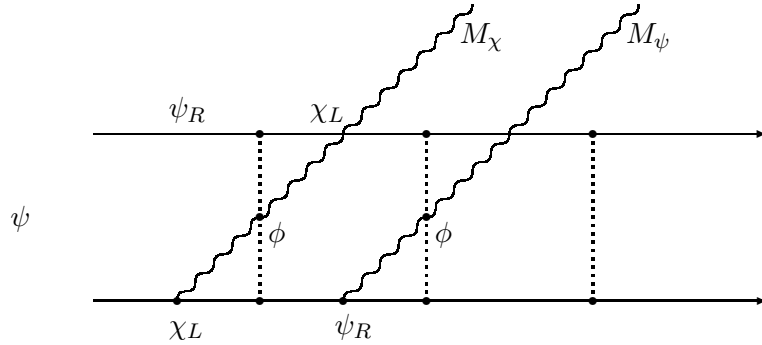


Fig.2: The constituents of a massive fermion. A massive fermion ψ consists of a left-handed massless fermion doublet state χ_L , a right-handed massless fermion singlet state ψ_R and a scalar doublet state ϕ . By the exchange of the scalar field ϕ the fermions χ_L and ψ_R are interchanged. The gauge potentials represented by the interaction matrices M_ψ and M_χ interact with ϕ and finally obtain a longitudinal component and mass.

The field ϕ introduced by (20) is consistent with the Lagrangian L_ϕ . However, for this field the Lagrangian \mathcal{L} does not appear acceptable for a theory of elementary particles, since for this field the Lagrangian L_Y is not a Lorentz scalar and L_ϕ has a mass term with the wrong sign. Hence, ϕ corresponds to a particle, which exists only during an exchange. In order to obtain correct Lagrangians and a renormalisable theory [5], the field ϕ has to be redefined.

Inserting (4), (6) and (20) the Lagrangian L_Y can be written as

$$L_Y = - \sum_i C_i H \bar{\psi}_i \left(\exp(i\Theta) \frac{1-\gamma^5}{2} + \exp(-i\Theta) \frac{1+\gamma^5}{2} \right) \psi_i. \quad (32)$$

A Lorentz scalar is only obtained if the γ^5 terms cancel. This is accomplished transforming ϕ into ϕ' via

$$\phi \rightarrow \phi' = \exp(i\Theta) \phi = \begin{pmatrix} a \\ b \end{pmatrix} H. \quad (33)$$

Thus, ϕ' obtains the same internal coordinates a and b as χ_{iL} . In order to leave the rest of the Lagrangian \mathcal{L} invariant, the gauge potentials W_μ^+, W_μ^- and Z_μ are simultaneously transformed by a gauge transformation

$$W_\mu^+ \rightarrow W_\mu^+ + \frac{1}{g} \partial_\mu \theta^+, \quad W_\mu^- \rightarrow W_\mu^- + \frac{1}{g} \partial_\mu \theta^-, \quad Z_\mu \rightarrow Z_\mu + \frac{1}{g} \partial_\mu \theta^0. \quad (34)$$

The internal degrees of freedom of ϕ represented by the Goldstone boson are thus transferred to the gauge potentials, providing the gauge potentials with a third component.

Together with the third component the gauge potentials must acquire mass. This is accomplished by the particular structure of L_ϕ . The Lagrangian L_ϕ consists of a kinetic energy density T_ϕ and a potential energy density V_ϕ

$$L_\phi = \frac{1}{2} (T_\phi - V_\phi) , \quad \text{where} \quad V_\phi = -\frac{m_H^2}{2} \phi^\dagger \phi + \frac{m_H^2}{4v^2} (\phi^\dagger \phi)^2. \quad (35)$$

The $(\phi^\dagger \phi)^2$ term represents a self-interaction. The vacuum state of ϕ is the field ϕ_0 for which V_ϕ has its minimum. With the choice of the parameters of L_ϕ the minimum of V_ϕ , for which

$$0 = \frac{dV_\phi}{d\phi} = -\frac{m_H^2}{2} \phi_0^\dagger + \frac{m_H^2}{2v^2} \phi_0^\dagger (\phi_0^\dagger \phi_0), \quad \text{is at} \quad \phi_0^\dagger \phi_0 = v^2. \quad (36)$$

Thus, the vacuum state ϕ'_0 for the transformed field ϕ' is obtained replacing H by v

$$\phi'_0 = \begin{pmatrix} a \\ b \end{pmatrix} v, \quad (37)$$

and the field ϕ for a particle state ϕ' created out of its vacuum state ϕ'_0 is given by

$$\phi = \phi'_0 + \phi' = \begin{pmatrix} a \\ b \end{pmatrix} (v + H). \quad (38)$$

The Lagrangians L_Y and L_ϕ imply that the general doublet field ϕ defined by (20) has to be redefined by this particular field ϕ . Inserting the expression (38) for the field ϕ into L_ϕ and gives with

$$(\partial_\mu - M_\phi)\phi = \begin{pmatrix} \left(\partial_\mu + i \frac{g}{2} \frac{Z_\mu}{\cos \theta_W} \right) a + i g \frac{W_\mu^+}{\sqrt{2}} b \\ i g \frac{W_\mu^-}{\sqrt{2}} a + \left(\partial_\mu - i \frac{g}{2} \frac{Z_\mu}{\cos \theta_W} \right) b \end{pmatrix} (v + H) \quad (39)$$

$$\begin{aligned} L_\phi &= \frac{1}{2} (\partial_\mu H)(\partial^\mu H) + \frac{g^2}{8} \left(2 W_\mu^- W^{\mu+} + \frac{Z_\mu Z^\mu}{\cos^2 \theta_W} \right) (\phi_0^\dagger \phi_0 + 2vH + H^2) \\ &+ \frac{m_H^2}{8v^2} (2v^2 (v + H)^2 - (v + H)^4). \end{aligned} \quad (40)$$

This Lagrangian consists of two terms, $L_\phi = L_H + L_{\phi_0}$, where L_H is the Lagrangian for the observable Higgs particle

$$\begin{aligned} L_H &= \frac{1}{2} (\partial_\mu H)(\partial^\mu H) - \frac{1}{2} m_H^2 H^2 + \frac{g^2}{8} \left(2 W_\mu^- W^{\mu+} + \frac{Z_\mu Z^\mu}{\cos^2 \theta_W} \right) (2vH + H^2) \\ &+ \frac{m_H^2}{8v^2} (v^4 - 4vH^3 - H^4) \end{aligned} \quad (41)$$

with the mass $m_H \approx 125 \text{ GeV}$ recently measured by the experiments ATLAS and CMS [6] at CERN.

The remaining term L_{ϕ_0} describes the interaction of the gauge potentials with the vacuum state ϕ_0 . Since the Proca Lagrangian for massive vector fields, corresponding to (29), has the form

$$L_W = \frac{1}{2} m_W^2 \mathbf{W}_\mu \mathbf{W}^\mu - \frac{1}{4} \mathbf{W}_{\mu\nu} \mathbf{W}^{\mu\nu}, \quad (42)$$

L_{ϕ_0} represents the required mass terms for the gauge potentials

$$\begin{aligned} L_{\phi_0} &= \frac{g^2}{8} \left(2 W_\mu^- W^{\mu+} + \frac{Z_\mu Z^\mu}{\cos^2 \theta_W} \right) \phi_0^\dagger \phi_0 \\ &= \frac{g^2 v^2}{8} \left(W_\mu^{+*} W^{\mu+} + W_\mu^{-*} W^{\mu-} + \frac{Z_\mu Z^\mu}{\cos^2 \theta_W} \right) \\ &= \frac{1}{2} m_W^2 \left(W_\mu^{+*} W^{\mu+} + W_\mu^{-*} W^{\mu-} + \frac{Z_\mu Z^\mu}{\cos^2 \theta_W} \right). \end{aligned} \quad (43)$$

Thus, the gauge potentials W_μ^+ , W_μ^- and Z_μ , which have obtained a longitudinal component, acquire mass, while A_μ remains massless. The gauge potential masses are

$$m_W = \frac{g}{2} v \quad \text{and} \quad m_Z = \frac{1}{\cos \theta_W} m_W. \quad (44)$$

The size of the constant v , which represents the electroweak mass scale, can be obtained via the Fermi coupling constant

$$G_F = \frac{\sqrt{2}}{8} \frac{g^2}{m_W^2} \approx 1.166 \cdot 10^{-5} \text{ GeV}^{-2} \quad (45)$$

resulting in

$$v = \frac{1}{\sqrt{\sqrt{2} G_F}} \approx 246 \text{ GeV}. \quad (46)$$

The gauge potentials rearrange, forming the observable physical fields to which the mass terms give mass. This rearrangement of the gauge potentials is a global unitary transformation

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} i & 0 & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} i & 0 & 0 \\ 0 & 0 & \cos \theta_W & -\sin \theta_W \\ 0 & 0 & \sin \theta_W & \cos \theta_W \end{pmatrix} \begin{pmatrix} W_\mu^1 \\ W_\mu^2 \\ W_\mu^3 \\ B_\mu \end{pmatrix} = \begin{pmatrix} W_\mu^+ \\ W_\mu^- \\ Z_\mu \\ A_\mu \end{pmatrix}, \quad (47)$$

leaving the Lagrangian for the gauge potentials unchanged

$$L_W + L_B = L_{W^+} + L_{W^-} + L_Z + L_A. \quad (48)$$

By the Yukawa interaction of the three elementary fields χ_{iL} , ψ_{iR} and ϕ a massive fermion ψ_i is formed. The two constituents χ_{iL} and ψ_{iR} turn into the two components of ψ_i . The symmetry of the massive fermion state ψ_i , which is also the symmetry of its components, is the symmetry which χ_{iL} and ψ_{iR} have in common. This is the symmetry of ψ_{iR} as a comparison of U_χ with U_ψ using (13) shows. Thus, ψ_{iR} is the right-handed component of ψ_i in agreement with its definition by (6). The corresponding left-handed component ψ_{iL} of ψ_i is due to (4) given by

$$\chi_{iL} = \begin{pmatrix} a \\ b \end{pmatrix} \psi_{iL}. \quad (49)$$

Using this relation and inserting the redefined field ϕ given by (38) into the Lagrangian L_Y results in a Lagrangian for a massive fermion ψ_i

$$\begin{aligned} L_Y &= - \sum_i C_i \left(\bar{\psi}_{iR} \phi^\dagger \chi_{iL} + \bar{\chi}_{iL} \phi \psi_{iR} \right) \\ &= - \sum_i C_i \left(\bar{\psi}_{iR} (v + H) (a^*, b^*) \begin{pmatrix} a \\ b \end{pmatrix} \psi_{iL} + \bar{\psi}_{iL} (a^*, b^*) \begin{pmatrix} a \\ b \end{pmatrix} (v + H) \psi_{iR} \right) \\ &= - \sum_i C_i (v + H) \left(\bar{\psi}_{iR} \psi_{iL} + \bar{\psi}_{iL} \psi_{iR} \right) \\ &= - \sum_i m_i \bar{\psi}_i \psi_i - \sum_i \frac{m_i}{v} \bar{\psi}_i H \psi_i. \end{aligned} \quad (50)$$

The first term is the mass term for a fermion ψ_i with the mass

$$m_i = C_i v, \quad (51)$$

and the second term describes the interaction of the Higgs particle H with a massive fermion ψ_i .

As required by neutrino oscillations, also the neutrinos have mass. Hence, the charges C_i are positive constants.

For the formation of a massive fermion ψ_i , the Lagrangian L_χ for the constituent χ_{iL} must be transformed into a Lagrangian for the left-handed component ψ_{iL} . Using the relation (49) gives

$$\begin{aligned} L_\chi &= \sum_i \bar{\chi}_{iL} i \gamma^\mu (\partial_\mu - M_\chi) \chi_{iL} \\ &= \sum_i \bar{\psi}_{iL} (a^*, b^*) i \gamma^\mu (\partial_\mu - M_\chi) \begin{pmatrix} a \\ b \end{pmatrix} \psi_{iL} \\ &= \sum_i \bar{\psi}_{iL} (a^*, b^*) \begin{pmatrix} a \\ b \end{pmatrix} i \gamma^\mu (\partial_\mu - M_\chi) \psi_{iL} \\ &= \sum_i \bar{\psi}_{iL} i \gamma^\mu (\partial_\mu - M_\chi) \psi_{iL}. \end{aligned} \quad (52)$$

Thus, the Lagrangians L_ψ and L_χ can be expressed in terms of the massive fermion ψ_i formed by the components ψ_{iR} and ψ_{iL} . In addition, the interaction matrices M given by (31) are inserted

$$\begin{aligned}
L_\psi &= \sum_i \bar{\psi}_i \mathrm{i} \gamma^\mu \frac{1+\gamma^5}{2} \left(\partial_\mu + \mathrm{i} q_i (A_\mu - \tan \theta_W Z_\mu) \right) \psi_i, \\
L_\chi &= \sum_i \bar{\psi}_i \mathrm{i} \gamma^\mu \frac{1-\gamma^5}{2} \left(\partial_\mu + \mathrm{i} q_i (A_\mu - \tan \theta_W Z_\mu) \right) \psi_i \\
&\quad + \sum_i \bar{\psi}_i \mathrm{i} \gamma^\mu \frac{1-\gamma^5}{2} \mathrm{i} \left(\frac{g}{\sqrt{2}} (T_+ W_\mu^+ + T_- W_\mu^-) + \frac{g t_3}{\cos \theta_W} Z_\mu \right) \psi_i. \quad (53)
\end{aligned}$$

Collecting the terms of the Lagrangian \mathcal{L} , using (50), (53), (41), (43) and (48), the complete Lagrangian \mathcal{L} may be expressed in terms of the observable physical fields

$$\begin{aligned}
\mathcal{L} &= \sum_i \bar{\psi}_i \left(\mathrm{i} \gamma^\mu \partial_\mu - m_i - \frac{m_i}{v} H \right) \psi_i \\
&\quad - \sum_i \bar{\psi}_i \gamma^\mu \frac{1-\gamma^5}{2} \left(\frac{g}{\sqrt{2}} (T_+ W_\mu^+ + T_- W_\mu^-) + \frac{g t_3}{\cos \theta_W} Z_\mu \right) \psi_i \\
&\quad - \sum_i \bar{\psi}_i \gamma^\mu q_i (A_\mu - \tan \theta_W Z_\mu) \psi_i \\
&\quad + \frac{1}{2} (\partial_\mu H) (\partial^\mu H) - \frac{1}{2} m_H^2 H^2 \\
&\quad + \frac{1}{8} g^2 \left(2 W_\mu^- W^{\mu+} + \frac{Z_\mu Z^\mu}{\cos^2 \theta_W} \right) (2vH + H^2) + \frac{1}{8} \frac{m_H^2}{v^2} (v^4 - 4vH^3 - H^4) \\
&\quad + \frac{1}{2} (2m_W^2 W_\mu^- W^{\mu+} + m_Z^2 Z_\mu Z^\mu) \\
&\quad - \frac{1}{4} \mathbf{W}_{\mu\nu} \mathbf{W}^{\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}. \quad (54)
\end{aligned}$$

This is the well established Lagrangian of the standard model [3]. Therefore, the original Lagrangian \mathcal{L} introduced by (24) to (31) is valid. The choice of the elementary fields, their charges, their Lagrangians and their rearrangement, forming massive physical fields, as presented, is thus justified.

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Appendix

Basics of Yang-Mills theory [7]

One internal dimension

A complex particle field $\Psi = \Psi(\exp(-i\varphi))$ undergoes a $U(1)$ interaction, transforming the field Ψ into the field Ψ'

$$\Psi \rightarrow \Psi' = U_0 \Psi, \quad (55)$$

where U_0 is a unitary transformation given by

$$U_0 = \exp\left(-i \int_{x^\mu}^{x'^\mu} g_0 B_\mu dx^\mu\right). \quad (56)$$

When a complex field carrying the $U(1)$ charge g_0 moves from space-time point x^μ to space-time point x'^μ interacting with a real, massless gauge potential B_μ , the internal variable φ is changed by $\Delta\varphi$. Simultaneously, the potential B_μ undergoes a gauge transformation

$$B_\mu \rightarrow B_\mu - \frac{1}{g_0} \partial_\mu(\Delta\varphi) \quad (57)$$

so that the total Lagrangian remains invariant under the interaction. The momentum operator for the free field $i\partial_\mu$ due to the interaction gets the eigenvalue $p_\mu + iU_0^{-1}\partial_\mu U_0$. Therefore, the momentum operator has to be modified by the replacement

$$\partial_\mu \rightarrow \partial_\mu - U_0^{-1}\partial_\mu U_0. \quad (58)$$

The field strength $F_{\mu\nu}$ created by the $U(1)$ potential B_μ is

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \quad (59)$$

It can be shown that the total $U(1)$ charge is conserved.

Three internal dimensions

A particle field $\Psi = \Psi(\mathbf{r})$, being equivalent to a real 3-vector \mathbf{r} , undergoes a $SO(3)$ interaction. By the interaction, the vector \mathbf{r} is rotated, and Ψ is transformed into Ψ' via a unitary transformation U_R

$$\Psi' = U_R \Psi(\mathbf{r}) = \Psi(R\mathbf{r}), \quad (60)$$

where the rotation R is given by

$$R = \exp\left(i \int g \sum_{\ell} J_{\ell} W_{\mu}^{\ell} dx^{\mu}\right) \quad (61)$$

and the rotation axes are denoted by ℓ . The generators of the rotations J_{ℓ} are

$$J_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad J_2 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \quad J_3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (62)$$

The charge g is the universal $SO(3)$ charge, and the gauge potentials W_μ^ℓ form a real 3-vector \mathbf{W}_μ of potentials. Moreover, the field strength $\mathbf{W}_{\mu\nu}$ created by the potential \mathbf{W}_μ is, after some algebra

$$\mathbf{W}_{\mu\nu} = \partial_\mu \mathbf{W}_\nu - \partial_\nu \mathbf{W}_\mu + g \mathbf{W}_\mu \times \mathbf{W}_\nu. \quad (63)$$

Four internal dimensions

A doublet of particle fields is represented by $\Psi = \Psi_0 \begin{pmatrix} a \\ b \end{pmatrix}$, where a and b are complex numbers with $a^*a + b^*b = 1$.

A complex 2-spinor $\begin{pmatrix} a \\ b \end{pmatrix}$ forms a real 4-vector $r_\nu = (a^*, b^*) \sigma_\nu \begin{pmatrix} a \\ b \end{pmatrix}$ with the Pauli matrices σ_ν

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (64)$$

Since the doublet is equivalent to a 3-vector, it undergoes an interaction transformation U_R , resulting from a rotation R of the 3-vector part \mathbf{r} of r_ν

$$(a^*, b^*) U_R^\dagger \sigma_\nu U_R \begin{pmatrix} a \\ b \end{pmatrix} = R \left((a^*, b^*) \sigma_\nu \begin{pmatrix} a \\ b \end{pmatrix} \right), \quad \nu = 1, 2, 3. \quad (65)$$

This equation is solved by

$$U_R = \exp \left(\pm i \int g \sum_\ell \frac{\sigma_\ell}{2} W_\mu^\ell dx^\mu \right). \quad (66)$$

The interaction transformation U_R of the doublet is hence the $SU(2)$ transformation, where the charge g is the universal $SO(3)$ charge.

To the $SU(2)$ interaction, a $U(1)$ interaction can be added, using the not yet used linearly independent σ matrix σ_0 as generator. In this case, the total interaction U of a doublet is the $SU(2) \times U(1)$ interaction, where B_μ is orthogonal to the W_μ potentials

$$\begin{aligned} U &= \exp \left(-i \int \left(g \sum_\ell \frac{\sigma_\ell}{2} W_\mu^\ell + g' y \sigma_0 B_\mu \right) dx^\mu \right) \\ &= \exp \left(-i \int \begin{pmatrix} g' y B_\mu + \frac{g}{2} W_\mu^3 & \frac{g}{2} (W_\mu^1 - i W_\mu^2) \\ \frac{g}{2} (W_\mu^1 + i W_\mu^2) & g' y B_\mu - \frac{g}{2} W_\mu^3 \end{pmatrix} dx^\mu \right). \end{aligned} \quad (67)$$

For convenience the $U(1)$ charge $g'y$ of the doublet is written as two factors, where g' serves as the $U(1)$ unit charge, and y is called the weak hypercharge.

The potentials implied by the $SU(2)$ interaction form an isospinor \mathcal{W} containing complex, charged fields

$$\mathcal{W} = \begin{pmatrix} W^+ \\ W^0 \\ W^- \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(W^1 - \mathrm{i} W^2) \\ W^3 \\ \frac{1}{\sqrt{2}}(W^1 + \mathrm{i} W^2) \end{pmatrix}, \quad \mathcal{W}^\dagger \mathcal{W} = \mathbf{W}^2. \quad (68)$$

The $SU(2) \times U(1)$ interaction transformation of a doublet can be rewritten in terms of the charged potentials introducing the operators

$$T_\pm = \frac{1}{2}(\sigma_1 \pm \mathrm{i} \sigma_2), \quad T_3 = \frac{1}{2}\sigma_3, \quad \text{and} \quad Y = y \sigma_0, \quad (69)$$

resulting in

$$U = \exp \left(-\mathrm{i} \int \left(g \left(\frac{1}{\sqrt{2}}(T_+ W_\mu^+ + T_- W_\mu^-) + T_3 W_\mu^3 \right) + g' Y B_\mu \right) \mathrm{d}x^\mu \right). \quad (70)$$

By an interaction, the doublet Ψ is transformed into Ψ' via

$$\Psi' = U \Psi. \quad (71)$$